SPECIAL CONTRIBUTIONS.

NOTE UPON ECONOMICAL SHAPES FOR CUTTING ENVELOPES OF BALLOONS.

By C. F. MARVIN, Professor of Meteorology, dated July 31, 1903.

Small balloons of rubber, silk, or paper ranging from 2 or 3 to 15 feet or more in diameter are frequently employed in connection with meteorological investigations now being conducted in the upper air, and it becomes a matter of importance to determine how the envelopes of such balloons can be formed in the most practical and economical manner. Suggestions made to me some months since by Professor Abbe, calling my attention to the greater economy afforded by certain unusual forms, has led me to examine the geometric principles involved in this problem. The results are of not a little interest to the geometer and moreover have a decided practical value in connection with the art of balloon construction, whether large or small.

Some of the conditions which are generally sought to be realized in balloon construction for serious purposes may be stated as follows:

(1) Minimum weight of the envelope consistent with strength and imperviousness to gases, etc.

(2) Maximum volume inclosed within minimum superficial envelope.

(3) Most equable distribution of the tensile stress to which all parts of the envelope are as a rule subjected.

(4) Facility and simplicity in computing and laying out an envelope of a given size, and ease in uniting the parts by cementing, sewing, or otherwise.

The selection of the material for the envelope practically decides the case under (1).

Any other form than a sphere involves a larger amount of surface and therefore greater weight in the envelope for the same volume inclosed.

The most equable distribution of superficial stresses is also realized in a sphere, or, at least, in a shape with hemispherical top, as the stress is greatest in this region. The spherical form therefore best satisfies the conditions under both (2) and (3).

The customary methods of forming the envelope of a balloon of approximately spherical shape consist in cutting the fabric of the envelope into a sufficient number of relatively narrow so-called *gores* of the proper form, and cementing or sewing these together along the edges. The seams in this case form meridional lines over the sphere, as indicated in fig. 1.

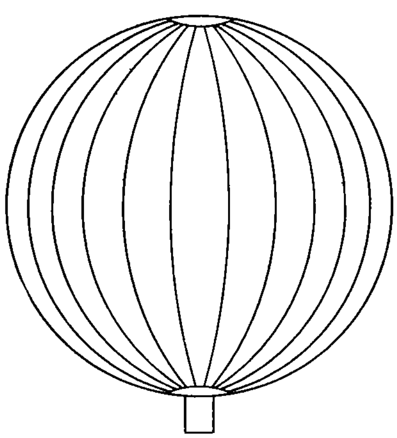
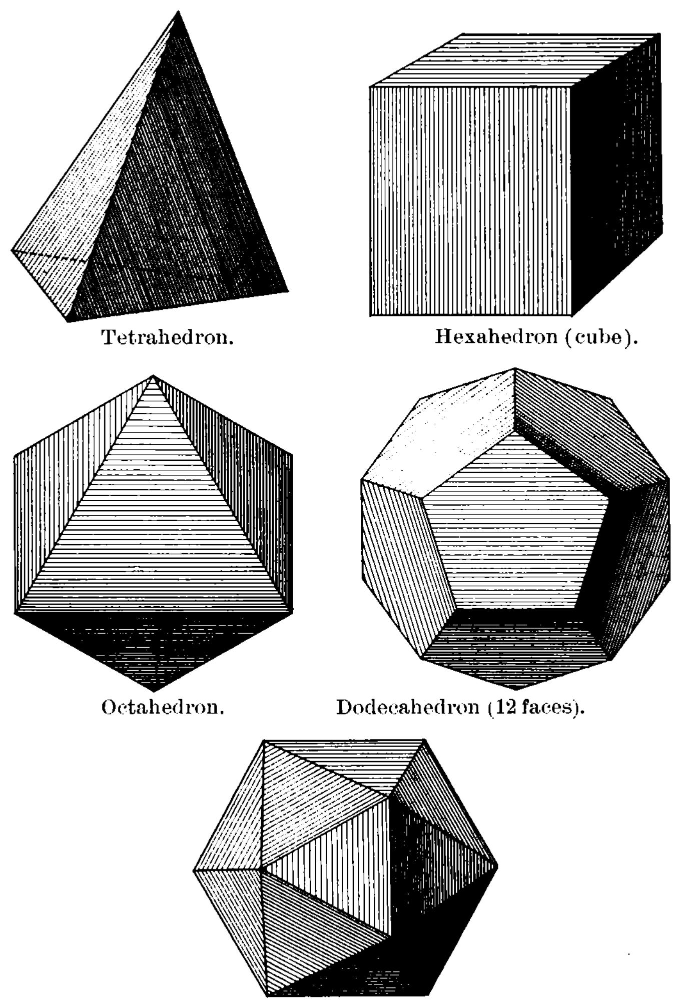


FIG. 1.

Gores of this character can not be cut without a very considerable waste, which is a matter of serious moment in the case of such valuable material as silk, rubber, etc. The length of a gore on a sphere of radius R will be $l = \pi R$, and if there are n gores the area of each, exclusive of overlap at seams, will be $1/n$th that of the spherical surface $= \frac{4\pi R^2}{n}$. So, likewise, the the width of the material from which the gores are cut must be $\frac{2\pi R}{n}$. Hence, the area of a band sufficient to form a gore must be $\frac{2\pi^2 R^2}{n}$ and the percentage of the band usefully employed $= \frac{2}{\pi} = 64$ per cent. In other words the material wasted in cutting gores amounts to $\frac{36}{64} = 57$ per cent of the surface of the sphere formed. This is obviously a very wasteful plan of construction, and while a small saving can be effected by overlapping the narrow points of the gores, yet this is only a partial remedy and not so satisfactory as other methods we shall describe presently.

It is interesting at this point to inquire into the suitability for this purpose of the *regular* or other polyhedral figures geometry has to offer. As is well known there are only five *regular* solids. These are shown in fig. 2.



Tetrahedron. Hexahedron (cube). Octahedron. Dodecahedron (12 faces). Icosahedron (20 faces).

FIG. 2.

The faces of these forms can be cut out with great economy, except perhaps in the case of the pentagon, but it is obvious that such figures as the cube, or even the octahedron, with eight sides, are unsuitable as a gas envelope, principally because of the unequal stresses over the different portions of the envelope. In fact, even the dodecahedron and the 20-sided solids are almost equally unavailable, owing to their relatively wide deviation from a sphere or figure of revolution.

Passing to the infinite number of possible *irregular* solids we may simply say that these, if not otherwise objectionable and unsuited, so far fail to satisfy condition (4) above as to be unavailable.

We find therefore that those methods of forming a gas envelope which aim to approximate more or less closely to a sphere prove to yield by far the most satisfactory results.

Zonal subdivision.—The peculiar style of spherical covering familiarly seen on "baseballs" can be formed with very little waste of material, as will be seen from the following description of a method of laying out a spherical envelope of this character. For this purpose four points are imagined to be located on the surface of the desired sphere so as to constitute the polar extremities of two axes at right angles to each other. One pair of poles is shown at p_1 and p_2, fig. 3.

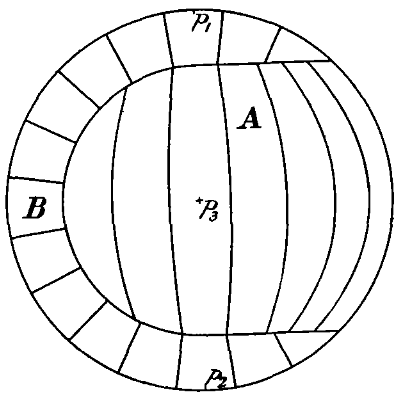


FIG. 3.

The other pair both appear projected at p_3. Now 4 semicircles, at latitude 45°, can be so drawn from these polar points and connected at points of tangency as to divide the spherical surface into two exactly equal and similar incomplete zonal bands or fields, *A B*, which interengage each other in the manner shown in fig. 3.

The fields may now be subdivided into any necessary number of meridional gores in the manner indicated.

Fifteen gores in each zonal field is regarded as a sufficient subdivision for a sphere about 10 feet in diameter, and one such subdivided zone is shown developed in fig. 4.

The distinctive advantage of this manner of laying out the envelope is found in the fact that with a few exceptions the gores are exactly alike throughout and have a symmetrical and nearly rectangular outline.

Fig. 5 shows how the unsymmetrical gores can be cut out from a band of material with minimum waste.

The coefficient of economy in this case can be computed in the following manner. With 15 gores to the zone the material from which the gores are cut must, for a sphere of radius *R*, have a width given by the equation:

$$w = \frac{\pi R}{10}.$$

The length of the long gore must be

$$l = \frac{\pi R}{2}.$$

The graphical construction in figs. 4 and 5, when made to scale, suffice to easily determine the relative amounts of material in the short gores; that is to say, upon measurement the band in fig. 5 will be found to aggregate 3.2 times the length of a normal gore. Consequently, the area of the material required to form the whole zonal field is $14.2\,\frac{\pi^2 R^2}{20}$. The spherical area of the zone is $2\pi R^2$; hence, the coefficient expressing the economy is $\frac{40}{14.2\pi} = 89$ per cent.

In this computation I have not included overlaps, which are necessary at the seams, for the reason that while this material is not directly used to form the spherical surface, yet it is necessary and useful and ought not to be computed with the waste.

From the above it is seen that only 11 per cent of the fabric is necessarily wasted, or that the waste amounts to 12 per cent of the spherical surface. This result is in marked contrast to the meridional gore construction, in which, as we have seen, 36 per cent of the material or 57 per cent of the spherical surface is wasted.

One unique feature of the zonal construction is that the zones unite without the formation of any congruent angles. The bounding lines of the zones unite in a continuous closed seam. This may not be so convenient, perhaps, for the insertion of an outlet pipe or neck for the envelope, but is, nevertheless, not of serious consequence.

Quadratic subdivision.—Another equally economical and simple plan of subdivision of the surface, and one recently employed in the construction of a large balloon for the Aeronautical Society of Munich, was described, with other interesting forms, by Dr. Finsterwalder in the Illustrirte Aeronautische Mittheilungen, October, 1902, p. 155.

The development of this subdivision and its geometric characteristics will be understood from what follows:

A cube is imagined to be inscribed within the desired sphere, and the points at which its corners touch the surface are connected by great circles; in other words, the edges of the cube

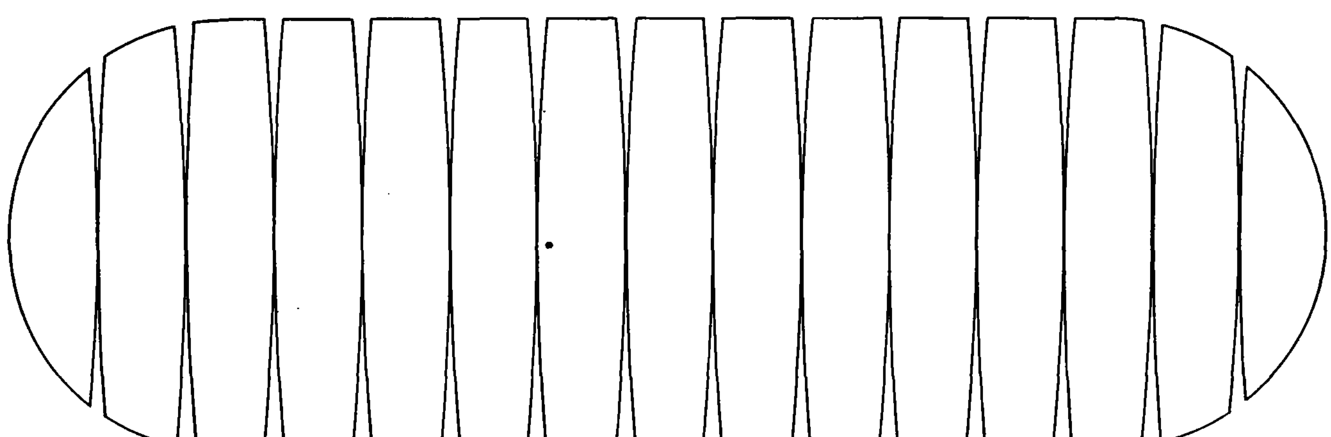


FIG. 4.

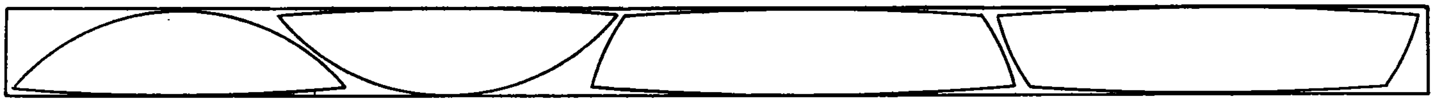


FIG. 5.

are projected upon the sphere by great circle planes, thus subdividing its surface into 6 equal and similar quadratic fields. These are shown in one view in fig. 6, in which each field is

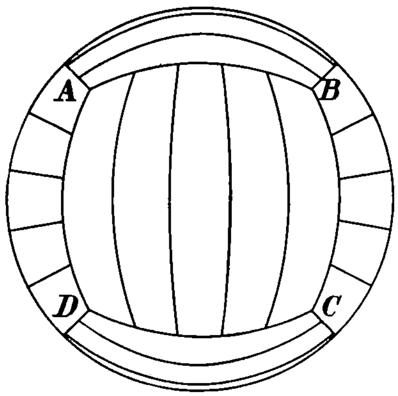


FIG. 6.

subdivided into 5 meridional gores which are very similar in general extent to the gores adopted above for the zonal subdivision. The extreme length of a gore is the length of the longest meridional line through it, and this is a maximum in the middle of a field where the length is one-quarter the circumference of the sphere $= \frac{\pi R}{2}$. The semi-length l of a meridian, in degrees intercepted between the boundaries of a field at any point having a longitude, a, measured from the central meridian, is given by the equation—

$$\tan l = \cos a. \qquad (1)$$

If n is the number of gores in a field, then the width of the gores in degrees is, $b° = \frac{90}{n}$.

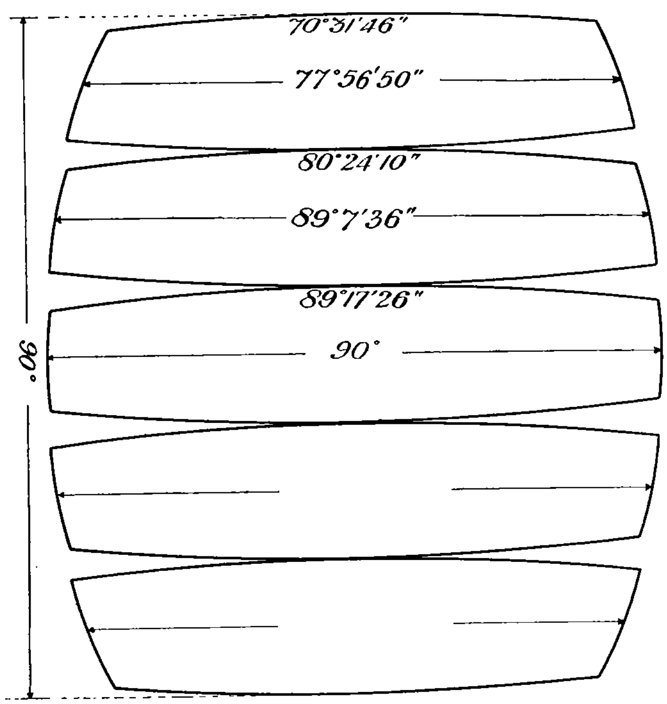


FIG. 7.

In the present case, with $n = 5$, $b° = 18$, and the meridional edges and central lines of the gores will have longitudes of 9°, 18°, 27°, 36°, and 45°, from which the lengths may easily be computed, and are entered in fig. 7.

In cutting out these gores the sloping-ended ones may be permitted to overlap in the manner already shown in fig. 5, the greatest economy being realized when a large number of the same kind of gores are cut from a continuous band. A saving of about 6 per cent can be effected in this way, for example, if the 12 small bands are cut together. If l is the length of the central line of a sloping-ended band and l_1 the length of the long edge, then the aggregate running length required to cut out n bands will be very nearly $nl + (l_1 - l)$. If many balloons are made n becomes a large number and the quantity $(l_1 - l)$ becomes negligible.

From these considerations the aggregate length of material necessary to cut the 30 gores required in the present case will be $\frac{14.047\ \pi^2 R^2}{10}$, and the width, not allowing for overlap at seams, will be $\frac{\pi R}{10}$.

$$\therefore \text{ area of material consumed} = \frac{14.047\ \pi^2 R^2}{10}.$$

$$\text{The surface of the sphere} = 4\pi R^2,$$

$$\text{and coefficient of economy} = \frac{40}{14.05\ \pi} = 90.6 \text{ per cent.}$$

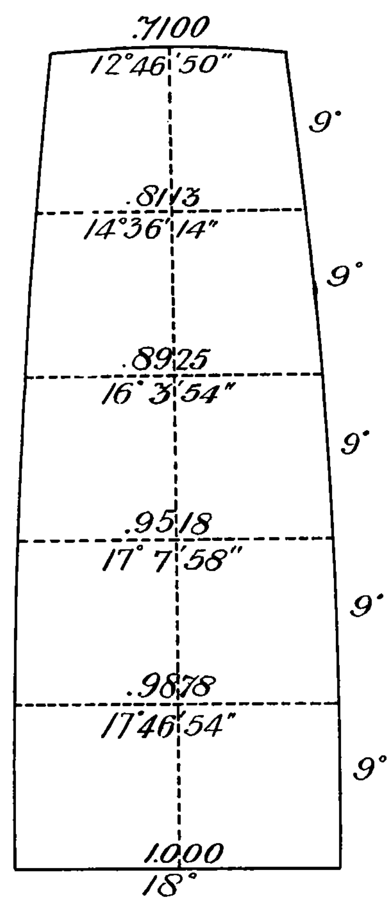


FIG. 8.

This shows a slightly higher economy than the zonal form, partly due to the assumed more favorable cutting of the sloping-ended gores.

To deduce the width of a gore at any point, let b equal the semi-width corresponding to the angular distance a from the equator of the gore, and b_0 the semi-width of the gore, in degrees at the equator, then, for radius = 1, we have the equation:

$$\tan b = \cos a \tan b_0.$$

In the quadratic fields a may vary from 0° to 45°; taking successive values of a at intervals of 9°, and assuming 5 gores to the field, viz, $b_0 = 9°$, we get the series of relative widths of the gore at successive points shown in fig. 8.

The overlap at seams may be treated in the following manner: In the case of 5 gores to a field, as shown in the foregoing illustrations, there will be in all 32 seams. The lengths of these, which correspond to the length of the edges of the gores, are given in fig. 7 and the sum is found to be 14.648 πR. An allowance of $\frac{3}{4}$ of an inch for overlap seems adequate, and in a balloon 10 feet in diameter this corresponds to about 4 per cent of the width of a gore. This figure may be adopted as convenient; hence, the area of seams is found to be 4.6 per cent of the area of the sphere.

The foregoing numerical data give all that is required in laying out balloons by this plan. In the case of rubber balloons its extreme extensibility renders a large number of gores unnecessary and no doubt a very good balloon can be made simply by uniting the six quadratic fields without subdivision, if large enough sheets of rubber were easily to be had. The maximum size of rubber balloons required in meteorological work will scarcely exceed 6 feet in diameter. The quadratic fields in such a balloon would be 56 inches wide, which is doubtless a greater width of rubber than is easily available. The field can be divided in the middle, necessitating only 12 exactly similar gores to make the balloons each 28 inches wide. The waste in cutting in these cases is a little greater than with a larger number of subdivisions.

BENJAMIN THOMPSON—COUNT RUMFORD.

By Mr. DANIEL T. PIERCE, JR.

Born at Woburn, Mass., March 26, 1753. His father, Benjamin Thompson, and his mother, Ruth Simonds, came of the stock of the first colonists of Massachusetts Bay.

His education was obtained by his own efforts, seconded by instruction from his grandfather; he attended no school except the primary institutions of the town where he was born. "Before he was 14 he could calculate and trace rightly the elevation of a solar eclipse." (Bibliothèque Britannique.) A letter written by him in 1769 to a friend who had assisted him in his studies shows the trend of his mind at the age of 16:

> Please to give the direction of the Rays of Light from a Luminous Body to an Opake, and the Reflection of an Opake Body to another equally Dense and Opake; viz., the Direction of the Rays of the Luminous Body to that of the Opake, and the Direction of the Rays by Reflection to the other Opake Body. N. B. From the Sun to the Earth, Reflected to the Moon at an angle of 40 degrees.

His Tory sympathies caused him to flee to England at the outbreak of the Revolution. Became under-secretary for the colonies under Germaine. Later colonel of the King's American Dragoons, stationed on Long Island. Was proscribed by the New Hampshire alienation act of 1778. (In 1772 he had married at Rumford, now Concord, the widow of Colonel Rolfe.) Returned to England; made fellow of the Royal Society in 1779; knighted by George III in 1784. In the same year he received permission to travel on the Continent, where he met Charles Theodore, Elector of Bavaria, and was from that time to 1797 the dominant influence in the administration of the Electorate, privy counselor, and in command of the army. He continued his investigations in heat and light and invented many appliances for the more economic consumption of fuel, including a kitchen range in which it is claimed that dinner for 1000 persons could be cooked at a fuel expense of four and one-half pennies. Was made count of the Holy Roman Empire in 1791 by the Elector. His labors in the interest of the poor brought him great popularity. In 1795 a marble memorial was erected in his honor in the English Gardens (laid out by him) at Munich, bearing on one side a relief medallion in bronze; a replica of this was long ago made in ivory by an unknown artist and from that a photograph was made by Mr. Thomas B. Gardiner of Washington, D. C.

The ivory replica from which this photograph portrait is taken is now in the possession of Daniel Thompson Pierce, of Washington, D. C., descendant of Josiah Pierce, of Woburn, second husband of the mother of Count Rumford.

Benjamin Thompson—Count Rumford.

In 1796 he gave $5000 each to the Royal Society and the American Society of Arts and Sciences, founding in each case a prize for the most important discoveries in heat and light.

In 1799, the year of the publication of his voluminous essays, he established the Royal Institution of Great Britain. Faraday, who was director of the institution in 1825, accords to Rumford the title of discoverer of the law of the correlation of forces, which, in the words of the former, is the "Highest philosophical idea that the human mind has been able to grasp."

Rumford had intended that the Institution should be to a large extent devoted to industrial work and experimentation in directions which would be of practical value especially to the poor. Meeting opposition in carrying out plans to this end, he abandoned the Institution and went to France, where he married the widow of the famous chemist Lavoisier. He died at Auteuil, near Paris, August 21, 1814, at the age of 61.